FMB with rock compressibility - Saturation Equations

Revised November 2026. Eqs. 20 and 47 (average pressure) and Eqs. 55–57 (recovery factor) are corrected, and a few typos are fixed. Equation numbers are unchanged. The changes are listed in the revision notes at the end.

Suppose we have a reservoir with pressure-dependent rock compressibility $c_r(p)$, so that its pore volume changes with pressure. By definition

$$c_r(p) = \frac{1}{V_p}\frac{dV_p}{dp} \tag{1}$$

Separating variables and integrating we have

$$\int_{V_p}^{V_{pi}} \frac{dV_p}{V_p} = \int_{p}^{p_i} c_r(p)dp \tag{2}$$

or

$$\ln\left(\frac{V_{pi}}{V_p}\right) = \int_{p}^{p_i} c_r(p)dp \tag{3}$$

or

$$V_p = V_{pi}\exp\left(-\int_{p}^{p_i} c_r(p)dp\right) \tag{4}$$

Define the pore volume multiplier γ as

$$\gamma = \exp\left(-\int_{p}^{p_i} c_r(p)dp\right) \tag{5}$$

Our material balance equations become

$$\begin{aligned} M_o(t) &= 5.615\rho_{osc}N_p \\ &= 5.615\rho_{osc}V_{pi}\left(\zeta_{oi} - \gamma\left(\frac{\bar{S}_o}{\bar{B}_o} + \frac{\bar{r}_v\bar{S}_g}{\bar{B}_g}\right)\right) \end{aligned} \tag{6}$$

$$\begin{aligned} M_g(t) &= 1000\rho_{gsc}G_p \\ &= 1000\rho_{gsc}V_{pi}\left(\zeta_{gi} - \gamma\left(\frac{\bar{R}_s\bar{S}_o}{\bar{B}_o} + \frac{\bar{S}_g}{\bar{B}_g}\right)\right) \end{aligned} \tag{7}$$

$$\begin{aligned} M_w(t) &= 5.615\rho_{wsc}W_p \\ &= 5.615\rho_{wsc}V_{pi}\left(\zeta_{wi} - \gamma\frac{\bar{S}_w}{\bar{B}_w}\right) \end{aligned} \tag{8}$$

where

$$\zeta_{oi} \equiv \frac{S_{oi}}{B_{oi}} + \frac{r_{vi}S_{gi}}{B_{gi}} \tag{9}$$

$$\zeta_{gi} \equiv \frac{R_{si} S_{oi}}{B_{oi}} + \frac{S_{gi}}{B_{gi}} \tag{10}$$

and

$$\zeta_{wi} \equiv \frac{S_{wi}}{B_{wi}} \tag{11}$$

and the total mass withdrawal at any time is given by

$$M_T(t) = M_o(t) + M_g(t) + M_w(t) \tag{12}$$

The cumulative fluid ratio relationships become

$$\text{OGR}\ (t) = \frac{\zeta_{oi} - \gamma\left[\frac{S_o}{B_o} + \frac{r_v S_g}{B_g}\right]_{well}}{\zeta_{gi} - \gamma\left[\frac{R_s S_o}{B_o} + \frac{S_g}{B_g}\right]_{well}} \tag{13}$$

$$\text{WGR}\ (t) = \frac{\zeta_{wi} - \gamma\left[\frac{S_w}{B_w}\right]_{well}}{\zeta_{gi} - \gamma\left[\frac{R_s S_o}{B_o} + \frac{S_g}{B_g}\right]_{well}} \tag{14}$$

Solving Eqs. 13 and 14 for the phase saturations, we have

$$S_o = -\frac{B_o\big(\Gamma_o + \zeta_{oi}\big(B_g + B_w\ \text{WGR}\big) - \zeta_{gi}\big(B_g\ \text{OGR} + B_w\ \text{WGR}\ r_v\big)\big)}{\gamma\big(B_g(\text{OGR}\ R_s - 1) - B_o(\text{OGR} - r_v) + B_w\ \text{WGR}\ (R_s r_v - 1)\big)} \tag{15}$$

where

$$\Gamma_o = (\gamma - \zeta_{wi} B_w)(\text{OGR} - r_v) \tag{16}$$

and

$$S_w = -\frac{B_w\big(\Gamma_w + \zeta_{wi}\big(B_g(1 - \text{OGR}\ R_s) + B_o(\text{OGR} - r_v)\big)\big)}{\gamma\big(B_g(\text{OGR}\ R_s - 1) - B_o(\text{OGR} - r_v) + B_w\ \text{WGR}\ (R_s r_v - 1)\big)} \tag{17}$$

where

$$\Gamma_w = \big(\gamma(1 - R_s r_v) + \zeta_{gi}\big(B_o r_v - B_g\big) + \zeta_{oi}\big(B_g R_s - B_o\big)\big)\ \text{WGR}$$

The MP FMB Equation is then

$$\frac{\beta_i - \gamma\beta_w}{\dot{m}} = \frac{1}{V_{pi}}\text{MBT}_M + \frac{1}{b_M} \tag{18}$$

Average Pressure Calculation

If we use Eqs. 15 and 17 evaluated at average reservoir pressure in any of the component mass balance equations (Eqs. 6 - 8) we will obtain an implicit equation in average pressure that can be solved iteratively. For the oil equation, material balance can be written as

$$N_i - N_p - \gamma V_{pi}\left(\frac{\bar{S}_o}{\bar{B}_o} + \frac{\bar{r}_v \bar{S}_g}{\bar{B}_g}\right) = 0 \tag{19}$$

where $N_i = V_{pi}\zeta_{oi}$, $G_i = V_{pi}\zeta_{gi}$ and $W_i = V_{pi}\zeta_{wi}$ are the initial oil, gas and water in place. Substituting in the saturation equations and setting the numerator of the resulting equation to zero, we obtain

$$\begin{aligned}&\big(N_p - N_i\big)\big(B_o - B_g R_s\big) + \big(G_p - G_i\big)\big(B_g - B_o r_v\big)\\ &\quad + \big(W_p B_w - W_i B_w + \gamma V_{pi}\big)(1 - R_s r_v) = 0\end{aligned} \tag{20}$$

where it is understood that all PVT properties and γ are evaluated at the average pressure. Eq. 20 follows more directly from Eqs. 6 - 8 and the saturation constraint. Dividing Eqs. 6 - 8 by their mass factors gives the oil, gas and water left in the reservoir,

$$N_i - N_p = \gamma V_{pi}\left(\frac{\bar{S}_o}{\bar{B}_o} + \frac{\bar{r}_v \bar{S}_g}{\bar{B}_g}\right), \quad G_i - G_p = \gamma V_{pi}\left(\frac{\bar{R}_s \bar{S}_o}{\bar{B}_o} + \frac{\bar{S}_g}{\bar{B}_g}\right), \quad W_i - W_p = \gamma V_{pi}\frac{\bar{S}_w}{\bar{B}_w}$$

Solving the first two for $\bar{S}_o/\bar{B}_o$ and $\bar{S}_g/\bar{B}_g$ and inserting the three saturations in $\bar{S}_o + \bar{S}_g + \bar{S}_w = 1$ gives

$$\bar{B}_o \frac{\big(N_i - N_p\big) - \bar{r}_v\big(G_i - G_p\big)}{1 - \bar{R}_s \bar{r}_v} + \bar{B}_g \frac{\big(G_i - G_p\big) - \bar{R}_s\big(N_i - N_p\big)}{1 - \bar{R}_s \bar{r}_v} + \bar{B}_w\big(W_i - W_p\big) = \gamma V_{pi}$$

which, multiplied by $-\big(1 - \bar{R}_s \bar{r}_v\big)$, is Eq. 20: the oil-phase, gas-phase and water volumes left in the reservoir fill the compacted pore volume γV_{pi}. Rock compaction reduces the pore volume; the oil, gas and water in place change only through production.

Quadratic Gas Rate Prediction Formula

If we consider 1-day timesteps, the cumulative producing Oil-Gas Ratio (OGR) at day $n+1$ is given by:

$$\mathrm{OGR}^{n+1} = \frac{N_p^{n+1}}{G_p^{n+1}} = \frac{N_p^n + \mathrm{ogr}^{n+1} q_g^{n+1}}{G_p^n + q_g^{n+1}} = \frac{\zeta_{oi} - \gamma\left[\frac{S_o^{n+1}}{B_o^{n+1}} + \frac{r_v^{n+1} S_g^{n+1}}{B_g^{n+1}}\right]}{\zeta_{gi} - \gamma\left[\frac{R_s^{n+1} S_o^{n+1}}{B_o^{n+1}} + \frac{S_g^{n+1}}{B_g^{n+1}}\right]} \tag{21}$$

where ogr^{n+1} is the instantaneous producing oil-gas ratio at time t^{n+1}. Similarly, the cumulative producing Water-Gas Ratio at day $n+1$ is given by:

$$\mathrm{WGR}^{n+1} = \frac{W_p^{n+1}}{G_p^{n+1}} = \frac{W_p^n + \mathrm{wgr}^{n+1} q_g^{n+1}}{G_p^n + q_g^{n+1}} = \frac{\zeta_{wi} - \gamma\left[\frac{S_w^{n+1}}{B_w^{n+1}}\right]}{\zeta_{gi} - \gamma\left[\frac{R_s^{n+1} S_o^{n+1}}{B_o^{n+1}} + \frac{S_g^{n+1}}{B_g^{n+1}}\right]} \tag{22}$$

where wgr^{n+1} is the instantaneous producing water-gas ratio at time t^{n+1}. In Eqs. 21 and 22 it is understood that all PVT properties and saturations are evaluated at sandface conditions. Using the saturation constraint

$$S_o^{n+1} + S_g^{n+1} + S_w^{n+1} = 1 \tag{23}$$

we can solve Eqs. 21 and 22 for oil and water saturations in terms of the new gas rate as

$$S_o^{n+1} = \frac{A_o q_g^{n+1} + C_o}{A_d q_g^{n+1} + C_d} \tag{24}$$

and

$$S_w^{n+1} = \frac{A_w q_g^{n+1} + C_w}{A_d q_g^{n+1} + C_d} \tag{25}$$

where

$$\begin{aligned} A_o = B_o^{n+1}\{&\big(B_g^{n+1}\mathrm{ogr}^{n+1} + B_w^{n+1} r_v^{n+1}\mathrm{wgr}^{n+1}\big)\zeta_{gi} \\ &-\big(B_g^{n+1} + B_w^{n+1}\mathrm{wgr}^{n+1}\big)\zeta_{oi} \\ &+(\mathrm{ogr}^{n+1} - r_v^{n+1})(B_w^{n+1}\zeta_{wi} - \gamma)\} \end{aligned} \tag{26}$$

$$\begin{aligned} C_o = B_o^{n+1}\{&\big(B_g^{n+1} N_p^n + B_w^{n+1} r_v^{n+1} W_p^n\big)\zeta_{gi} \\ &-\big(B_g^{n+1} G_p^n + B_w^{n+1} W_p^n\big)\zeta_{oi} \\ &+\big(N_p^n - G_p^n r_v^{n+1}\big)(B_w^{n+1}\zeta_{wi} - \gamma)\} \end{aligned} \tag{27}$$

$$\begin{aligned} A_d = \gamma\big(&B_g^{n+1}(\mathrm{ogr}^{n+1} R_s^{n+1} - 1) \\ &+B_o^{n+1}(r_v^{n+1} - \mathrm{ogr}^{n+1}) + B_w^{n+1}(R_s^{n+1} r_v^{n+1} - 1)\mathrm{wgr}^{n+1}\big) \end{aligned} \tag{28}$$

$$\begin{aligned} C_d = \gamma\big(&N_p^n\big(B_g^{n+1} R_s^{n+1} - B_o^{n+1}\big) \\ &+G_p^n\big(B_o^{n+1} r_v^{n+1} - B_g^{n+1}\big) + B_w^{n+1}(R_s^{n+1} r_v^{n+1} - 1) W_p^n\big) \end{aligned} \tag{29}$$

$$\begin{aligned} A_w = B_w^{n+1}\{&\big(\big(B_g^{n+1} - B_o^{n+1} r_v^{n+1}\big)\zeta_{gi} \\ &+\big(B_o^{n+1} - B_g^{n+1} R_s^{n+1}\big)\zeta_{oi} - \gamma(1 - R_s^{n+1} r_v^{n+1})\big)\mathrm{wgr}^{n+1} \\ &+\big(\mathrm{ogr}^{n+1}\big(B_g^{n+1} R_s^{n+1} - B_o^{n+1}\big) + B_o^{n+1} r_v^{n+1} - B_g^{n+1}\big)\zeta_{wi}\} \end{aligned} \tag{30}$$

$$\begin{aligned} C_w = B_w^{n+1}\{&\big(\gamma(R_s^{n+1} r_v^{n+1} - 1) \\ &-\big(B_o^{n+1} r_v^{n+1} - B_g^{n+1}\big)\zeta_{gi} - \big(B_g^{n+1} R_s^{n+1} - B_o^{n+1}\big)\zeta_{oi}\big) W_p^n \\ &+\big(B_g^{n+1}\big(N_p^n R_s^{n+1} - G_p^n\big) - B_o^{n+1}\big(N_p^n - G_p^n r_v^{n+1}\big)\big)\zeta_{wi}\} \end{aligned} \tag{31}$$

Suppose that for a given reservoir we have a well established multiphase flowing material balance straight line,

$$\frac{\beta_i - \gamma\beta_w^{n+1}}{\dot{m}^{n+1}} = \frac{1}{V_{pi}}\mathrm{MBT}_M + \frac{1}{b_M} \tag{32}$$

Multiplying both sides of the equation evaluated at time t^{n+1} by the mass flow rate, we have

$$\beta_i - \gamma\beta_w^{n+1} = \frac{1}{V_{pi}} M_T^{n+1} + \frac{\dot{m}^{n+1}}{b_M} \tag{33}$$

The mass flow rate at time t^{n+1} is given by

$$\dot{m}^{n+1} = q_g^{n+1}\big(5.615\rho_{osc}\mathrm{ogr}^{n+1} + 1000\rho_{gsc} + 5.615\rho_{wsc}\mathrm{wgr}^{n+1}\big) \tag{34}$$

and the cumulative mass at time t^{n+1} is

$$M_T^{n+1} = M_T^n + \dot{m}^{n+1} \tag{35}$$

Let

$$\alpha_o = 5.615\rho_{osc} \tag{36}$$

$$\alpha_g = 1000\rho_{gsc} \tag{37}$$

$$\alpha_w = 5.615\rho_{wsc} \tag{38}$$

$$\alpha_c = \alpha_o \mathrm{ogr}^{n+1} + \alpha_g + \alpha_w \mathrm{wgr}^{n+1} \tag{39}$$

Eq. 33 can be written as

$$\beta_i - \frac{\gamma\left(D_q q_g^{n+1} + E\right)}{B_g^{n+1} B_o^{n+1} B_w^{n+1}\left(A_d q_g^{n+1} + C_d\right)} = \frac{M_T^n}{V_{pi}} + \alpha_c \left(\frac{1}{V_{pi}} + \frac{1}{b_M}\right) q_g^{n+1} \tag{40}$$

where

$$\begin{aligned} D_q = {} & A_o B_g^{n+1} B_w^{n+1}\left(\alpha_o + \alpha_g R_s^{n+1}\right) + \alpha_w A_w B_g^{n+1} B_o^{n+1} \\ & + (A_d - A_o - A_w) B_o^{n+1} B_w^{n+1}\left(\alpha_g + \alpha_o r_v^{n+1}\right) \end{aligned} \tag{41}$$

and

$$\begin{aligned} E = {} & B_g^{n+1}\left(\alpha_w B_o^{n+1} C_w + \left(\alpha_o + \alpha_g R_s^{n+1}\right) B_w^{n+1} C_o\right) \\ & + \left(\alpha_g + \alpha_o r_v^{n+1}\right) B_o^{n+1} B_w^{n+1} (C_d - C_o - C_w) \end{aligned} \tag{42}$$

Eq. 40 can be rearranged as a quadratic equation in q_g^{n+1}, i.e.,

$$a\left(q_g^{n+1}\right)^2 + b q_g^{n+1} + c = 0 \tag{43}$$

where

$$a = -\frac{A_d \alpha_c B_g^{n+1} B_o^{n+1} B_w^{n+1}\left(b_M + V_{pi}\right)}{b_M V_{pi}} \tag{44}$$

$$\begin{aligned} b = \gamma\Big(& \left((A_o + A_w - A_d) B_w^{n+1}\left(\alpha_g + \alpha_o r_v^{n+1}\right) - \alpha_w A_w B_g^{n+1}\right) B_o^{n+1} \\ & - A_o B_g^{n+1} B_w^{n+1}\left(\alpha_o + \alpha_g R_s^{n+1}\right)\Big) \\ & - B_g^{n+1} B_o^{n+1} B_w^{n+1}\left(\frac{\alpha_c C_d + A_d M_T^n}{V_{pi}} + \frac{\alpha_c C_d - A_d \beta_i b_M}{b_M}\right) \end{aligned} \tag{45}$$

and

$$\begin{aligned} c = -\gamma\Big(& \left(B_w^{n+1} C_o\left(\alpha_o + \alpha_g R_s^{n+1}\right) + \alpha_w B_o^{n+1} C_w\right) B_g^{n+1} \\ & + B_o^{n+1} B_w^{n+1} (C_d - C_o - C_w)\left(\alpha_g + \alpha_o r_v^{n+1}\right)\Big) \\ & + B_g^{n+1} B_o^{n+1} B_w^{n+1} C_d \frac{\beta_i V_{pi} - M_T^n}{V_{pi}} \end{aligned} \tag{46}$$

The positive root of this quadratic is the new predicted rate.

Recovery Factor Calculation

If we consider the average pressure formula, Eq. 20, we have

$$\begin{aligned} & \left(N_p - N_i\right)\left(B_o - B_g R_s\right) + \left(G_p - G_i\right)\left(B_g - B_o r_v\right) \\ & + \left(W_p B_w - W_i B_w + \gamma V_{pi}\right)\left(1 - R_s r_v\right) = 0 \end{aligned} \tag{47}$$

Define

$$RF_o = \frac{N_p}{N_i} \tag{48}$$

$$m = \frac{(G_i - N_i R_{si})B_{gi}}{(N_i - G_i r_{vi})B_{oi}} = \frac{S_{gi}}{S_{oi}} \tag{49}$$

so

$$\frac{G_i}{N_i} = \frac{mB_{oi} + B_{gi}R_{si}}{B_{gi} + mB_{oi}r_{vi}} \tag{50}$$

The saturation constraint can be written as

$$1 - S_{wi} - S_{oi}\left(1 + \frac{S_{gi}}{S_{oi}}\right) = 1 - S_{wi} - S_{oi}(1+m) = 0 \tag{51}$$

so

$$S_{oi} = \frac{1 - S_{wi}}{1 + m} \tag{52}$$

Note that

$$\frac{G_p}{N_i} = \frac{G_p}{N_p}\frac{N_p}{N_i} = \frac{RF_o}{\text{OGR}} \tag{53}$$

$$\frac{W_p}{N_i} = \frac{W_p}{G_p}\frac{G_p}{N_i} = \frac{\text{WGR } RF_o}{\text{OGR}} \tag{54}$$

By Eqs. 9 and 10, $(N_i - G_i r_{vi})B_{oi} = V_{pi}S_{oi}(1 - R_{si}r_{vi})$, and by Eq. 50, $1 - r_{vi}G_i/N_i = B_{gi}(1 - R_{si}r_{vi})/\big(B_{gi} + mB_{oi}r_{vi}\big)$. So

$$\begin{aligned}
\frac{W_i}{N_i} &= \frac{W_i B_{wi}}{(N_i - G_i r_{vi})B_{oi}} \frac{(N_i - G_i r_{vi})B_{oi}}{N_i B_{wi}} \\
&= \frac{S_{wi}}{S_{oi}(1 - R_{si}r_{vi})} \frac{\left(1 - \frac{G_i}{N_i}r_{vi}\right)B_{oi}}{B_{wi}} \\
&= \frac{S_{wi}B_{oi}B_{gi}}{S_{oi}B_{wi}\big(B_{gi} + mB_{oi}r_{vi}\big)}
\end{aligned} \tag{55}$$

$$\begin{aligned}
\frac{V_{pi}}{N_i} &= \frac{V_{pi}}{(N_i - G_i r_{vi})B_{oi}} \frac{(N_i - G_i r_{vi})B_{oi}}{N_i} \\
&= \frac{1}{S_{oi}(1 - R_{si}r_{vi})}\left(1 - \frac{G_i}{N_i}r_{vi}\right)B_{oi} \\
&= \frac{B_{oi}B_{gi}}{S_{oi}\big(B_{gi} + mB_{oi}r_{vi}\big)}
\end{aligned} \tag{56}$$

Dividing Eq. 47 by N_i and using the relationships in Eqs. 48 - 56, we have

$$
\begin{aligned}
&(RF_o - 1)\big(B_o - B_g R_s\big) + \left(\frac{RF_o}{\mathrm{OGR}} - \frac{mB_{oi} + B_{gi}R_{si}}{B_{gi} + mB_{oi}r_{vi}}\right)\big(B_g - B_o r_v\big) \\
&\quad + \left(\frac{\mathrm{WGR}\ RF_o}{\mathrm{OGR}} B_w + \frac{(1+m)B_{oi}B_{gi}}{(1-S_{wi})\big(B_{gi} + mB_{oi}r_{vi}\big)}\left(\gamma - \frac{S_{wi}}{B_{wi}}B_w\right)\right)(1 - R_s r_v) = 0
\end{aligned}
\tag{57}
$$

Revision notes (November 2026)

This revision corrects the following equations of the earlier version of this note. Equation numbers are unchanged.

Eqs. 20 and 47. The earlier average pressure equation was

$$
\begin{aligned}
&\big(N_p - \gamma N_i - (1-\gamma)G_i\ \mathrm{OGR}\big)\big(B_o - B_g R_s\big) + \big(G_p - G_i\big)\big(B_g - B_o r_v\big) \\
&\quad + \big(W_p B_w - (1-\gamma)G_i\ \mathrm{WGR}\ B_w + \gamma\big(V_{pi} - W_i B_w\big)\big)(1 - R_s r_v) = 0
\end{aligned}
$$

Its left side exceeds that of Eq. 20 by

$$
(1-\gamma)\big[(N_i - G_i\ \mathrm{OGR})\big(B_o - B_g R_s\big) + (W_i - G_i\ \mathrm{WGR})B_w(1 - R_s r_v)\big]
$$

so it holds only without rock compressibility ($\gamma = 1$), or when oil and water happen to be produced in their in-place ratios to gas. It applies γ to the initial oil and water in place as well as to the pore volume. Eqs. 13 - 19 are correct: substituting Eqs. 15 and 17 in Eq. 19 gives a fraction whose numerator is OGR times the left side of Eq. 20. The difference is largest just above the saturation pressure, where the reservoir is least compressible; on a black-oil example with $c_r = 4 \times 10^{-6}\ \mathrm{psi}^{-1}$ the earlier equation moved the average pressure by about 100 psi there.

Eqs. 55 and 56. The earlier forms took $(N_i - G_i r_{vi})B_{oi}$ as $V_{pi}S_{oi}$ and so missed the factor $1/(1 - R_{si}r_{vi})$. They hold only when $R_{si}r_{vi} = 0$.

Eq. 57. Besides inheriting the correction of Eq. 47, the earlier form wrote G_i/N_i as mB_{oi}/B_{gi} and used the earlier Eqs. 55 and 56, so it was Eq. 47 divided by N_i only when $R_{si} = r_{vi} = 0$. With $R_{si} = r_{vi} = 0$, $G_i/N_i = mB_{oi}/B_{gi}$ and $V_{pi}/N_i = (1+m)B_{oi}/(1 - S_{wi})$, and Eq. 57 then differs from the earlier form only in that γ multiplies the pore volume alone.

Typos. Eq. 8: ρ_{wsc} instead of ρ_{gsc} in the second line. Eq. 14: B_o instead of B in the denominator. Eq. 21: B_g^{n+1} instead of B_g in the numerator. Eqs. 34 and 39: the water term uses wgr^{n+1} instead of ogr^{n+1}. Eqs. 24 and 25 give the oil and water saturations (the text said oil and gas). Eqs. 32, 33 and 40 write the pore volume from the FMB slope as V_{pi}, as Eqs. 18 and 44 - 46 do.

Verification. The equations were checked symbolically with SymPy: Eqs. 15 and 17 solve Eqs. 13 and 14 with $S_o + S_g + S_w = 1$; Eq. 19 with Eqs. 15 and 17 gives Eq. 20; Eqs. 24 - 31 solve Eqs. 21 - 23; Eqs. 41 and 42 give the mixture density at the well, and Eqs. 43 - 46 the quadratic of Eq. 40; Eqs. 49 - 56 follow from Eqs. 9 - 11; and Eq. 57 is Eq. 47 divided by N_i for any R_{si} and r_{vi}.